

“Einstein’s Dream” – Quantum Mechanics as Theory of Classical Random Fields

Andrei Khrennikov

International Center for Mathematical Modeling
in Physics, Engineering, Economics, and Cognitive Science
Linnaeus University, Växjö-Kalmar, Sweden

March 3, 2013

Abstract

This is an introductory chapter of the book in progress on quantum foundations and incompleteness of quantum mechanics. Quantum mechanics is represented as statistical mechanics of classical fields.

Preface

This book is dedicated to Einstein’s vision of physics and specifically his hope for what quantum theory could and, in his view, should be. In particular, two of Einstein’s dreams about the future of quantum theory are realized in this book: a reduction of quantum randomness to classical ensemble randomness and the total elimination of particles from quantum mechanics (QM) – the creation of a field model of quantum phenomena. Thus, contrary to a number of the so-called no-go arguments and theorems advanced throughout the history of quantum theory (such as those of von Neumann, Kochen-Specker, and Bell), quantum probabilities and correlations can be described in a classical manner.

There is, however, a crucial proviso. While this book argues that QM can be interpreted as a form of classical statistical mechanics (CSM), this classical statistical theory is not that of particles, but of fields. This means that the mathematical formalism of QM must be

translated into the mathematical formalism of CSM on the infinite-dimensional phase space. The infinite dimension of the phase space of this translation is a price of classicality. From the mathematical viewpoint this price is very high, because in this case the theories of measure, dynamical systems, and distributions are essentially more complicated than in the case of the finite-dimensional phase space found in a CSM of particles. However, at the model level (similar to quantum information theory) one can proceed with the finite-dimensional phase space by approximating physical prequantum fields by vectors with finite number of coordinates. To simplify the presentation and by taking into account that usage of infinite-dimensional analysis (in the rigorous mathematical framework) is still not common in the quantum community, we present the basic constructions in finite dimensional (so to say n -qubits) Hilbert spaces. (Special sections are devoted to generalization to the case of fields.)

On the other hand, from the physical and philosophical viewpoints, considering QM as a CSM of fields can resolve the basic interpretational problems of QM. For example and in particular, quantum correlations of entangled systems can be reduced to correlations of classical random fields. From this perspective, quantum entanglement is not mysterious at all, since quantum correlations are no longer different from the classical ones.¹

The main difficulty is that the classical situation is very tricky by itself. All quantum correlations contain the irreducible contribution of a background field (vacuum fluctuations). Roughly speaking quantum systems are classical random signals that are measured against a sufficiently strong random background. The data of QM is a result of our ignorance concerning the contribution of this random background. Thus quantum probabilities and correlations are not simply classical quantities. They are obtained from classical quantities by means of

¹In fact, the situation is more complicated. Averages and correlations provided by the classical field theory are related to continuous signals, but the quantum ones are based on statistics of discrete clicks of detectors. However, it is possible to discretize continuous signals with the aid of the *threshold type detectors* and transform probabilistic quantities for continuous signals into statistics of discrete clicks which coincide with quantum ones. The main part of this book is devoted to representation of quantum averages and correlations with the aid of continuous random signals. Already this step is nontrivial both from physical and mathematical viewpoints. The corresponding discretization model will be presented, see also [263] for more details. We call this model *threshold detection model* (TSD).

a renormalization procedure: a subtraction of the contribution of the background field. Accordingly, the reduction of quantum randomness to classical is not totally straightforward. Nevertheless, it is possible. For example, the otherwise mysterious nature of the Heisenberg uncertainty principle can be resolved in the following way. Quantum dispersions are not simply classical dispersions, but the results of the subtraction of the dispersion of vacuum fluctuations. There is nothing mysterious in the fact that renormalized quantities satisfy this type of inequality. This is not so unusual from the viewpoint of the classical probability theory.

Already Max Planck emphasized the role of a random background field, the concept that was widely used in stochastic electrodynamics. (In his letter to Einstein he pointed out that spontaneous emission can be easily explained by taking into account the background field.) This field also plays an essential role in our model, CSM of classical fields, also termed as *prequantum classical statistical field theory* (PCSFT). This model is purely that of the field type. A classical random field is associated with each type of quantum particles. We have, for example, the electronic, neutronic, and protonic fields. The photonic field is simply the classical electromagnetic field of low intensity.

There is also a deep-going analogy between the present approach and the *classical theory of random signals*. Quantum measurements can be described as measurements for classical random signals with a noisy background. This analogy between QM and classical signal theory had been explored from the reverse perspective, for example, in using quantum information theory in the theory of classical Gaussian random signals. Indeed, restricting the present discussion to Gaussian signals alone would significantly simplify the presentation of PCSFT. However, the present book attempts to proceed by considering arbitrary random signals as much as possible.

I hope that the book will stimulate research that aims to demystify QM and to create a purely field model of quantum reality, and even to go beyond QM and find classical wave phenomena behind the basic laws of QM, such as Born's rule for probabilities. In PCSFT, prequantum random fields fluctuate on a time scale that is essentially finer than the time scale of quantum measurements. The fundamental question is whether this time scale is physically approachable remains open. In particular, if the prequantum time scale were the Planck scale, the PCSFT-level would be inapproachable. There would be no hope to monitor prequantum waves and show how the quantum statistics of

clicks of detectors is produced through interaction of such waves with the threshold-type (macroscopic) detectors. However, if the prequantum time scale is essentially coarser than the Planck scale, one might dream of finding experimental confirmation of derivability of quantum laws (which are fundamentally probabilistic) from behaviour of prequantum (random) waves. Either way, however, PCSFT provides an adequate theoretical model of reduction of QM to CSM of fields.

The present wave of interest in quantum foundations is caused by the tremendous development of quantum information science and its applications to quantum computing and quantum communication. Nowadays this interest even increases, because *it became clear that some of the difficulties encountered in realizations of quantum information processing are not simply technicalities, but instead have roots at the very fundamental level.* To solve such difficult problems, quantum theory has to be reconsidered. In particular, some prejudices must be discarded; first of all the prejudice on completeness of QM.

Växjö-Moscow-Tokyo

Andrei Khrennikov
2008-2012

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Introductory Chapter

1 Author's views on quantum foundations

This section is full of my reminiscences of meetings in Våxjö, philosophical and historical remarks on views of Hertz, Boltzmann, Planck, Einstein, Bohr, Schrödinger, von Neumann, De Broglie, von Mises, Lamb and Scully, Lande, Bohm, Mackey, Marshall and Brafford, Boyer, de la Pena and Cetto, Hiley, Emch, Cole, Ballentine, Vidman, Peres, Ohya, Accardi, Gill, Fuchs, Plotnitsky, Zeilinger, Aspect, Rauch, Weihs, Volovich, Holevo, Belavkin, Ozawa, De Muynck, De Baere, Elitzur, Peres, Greenberger, ... This section can hopefully be of interest as containing details of history of quantum foundations; or the reader can jump directly to Section 2 which contains a short introduction to my “beyond quantum model” – *prequantum classical statistical field theory* (PCSFT).

1.1 Debates in Våxjö

To better understand what this book is about, it may be useful to know a little bit about the author's views. In the quantum foundations community I am well known as the organizer of the series of conferences which have been held in small town Våxjö in the South-East part of Sweden. This town surrounded by woods and lakes is really a good place for contemplations of kinds, in my case, specifically, about quantum theory, one of the most exciting theories ever created. The theory is exciting not only because of its tremendous advances, but also because of its paradoxical claims and conclusions.

The series of Våxjö conferences on foundations of quantum mechanics (especially probabilistic foundations) combines two subseries: *Foundations of Probability and Physics*: 2000, 02, 04, 06, 08, 11 [127], [133], [141], [10], [7]; and *Quantum Theory: Reconsideration of Foundations*: 2001, 03, 05, 07, 09, 12 [130], [140], [8], [12], [182]. A new series *Advances in Quantum Theory* started in 2010. All the conferences have been notable not only for the original contributions but also for several exciting debates that took place there. These debates offered a great diversity of perspectives on foundations of quantum mechanics

(QM) and its future developments: from the orthodox Copenhagen interpretation (which rejects realism and causality), at one end of the spectrum, to more realistic views, as advocated by Einstein, at the other end.

During the last ten years I have been lucky to meet the world leading experts in quantum foundations and discuss with them the most intriguing problems. What surprised me (at least at the first conferences)? It was the huge diversity of opinions and views on the very fundamental and old problems. My expectation that by inviting great quantum gurus I can get clear answers was naive. The first conference, *Bohmian mechanics 2000*, was the total fiasco: two leading representatives of Bohmian school, Shelly Goldstein and Basil Hiley, presented two totally different interpretations of Bohmian mechanics. Finally, they accused each other in misunderstanding of Bohm's views (both had very close connections to David Bohm). My students whom I invited to learn Bohmian mechanics from its creators were really confused. The only useful information which I extracted from *Bohmian mechanics 2000* was that Bohmian mechanics does not give new experimental predictions comparing to conventional QM. Thus, although formally (mathematically) Bohmian mechanics provides a finer description of micro processes, it is impossible to design experiments which will distinguish Bohmian mechanics and QM.

Similar stories have repeated quite a few times with various fundamental problems. Only in some cases I was lucky to learn something. For example, I got the answer to the question: "What is crucial in quantum computing: superposition or entanglement?" I learned that superposition plays a subsidiary role, the crucial is entanglement; the classical wave computer (e.g., optical) cannot beat the classical digital computer. However, yet another simple question has never been clarified: "Do pure states provide better quantum computational resource than mixed?" Opinions of quantum computing gurus did not converge to the common point. And I can mention a series of similar questions, e.g., "How dangerous for quantum cryptography is detectors inefficiency?" People who spontaneously answered me that the impact of the detectors inefficiency can be easily taken into account, a few years later applied for grants to study this "very important problem".

I can mention a series of simple Văxjö-questions without answers, e.g.: "What is electron? What is the origin of discreteness of the electric charge? What is the essence of vacuum fluctuations? Can the mathematical formalism of QM be applied outside of physics, e.g., in

cognitive science?”

However, the most exciting spectacle started each time when the question of *interpretations of the wave function* attracted the attention. Finally, I understood that the number of different interpretations is in the best case equal to the number of participants. If you meet two people who say that they are advocates of, e.g., the Copenhagen interpretation of QM, ask them about the details. You will see immediately that their views on what is the Copenhagen interpretation can differ very much. The same is true for other interpretations. If two scientists tell that they are followers of Albert Einstein’s *ensemble interpretation*, ask them about the details... At one of the round tables (after two hours of debates with opinions for and against completeness of QM) we had decided to vote on this problem. Incompleteness advocates have won, but only because a few advocates of completeness voted for incompleteness. The situation is really disappointing: the basic notion of QM has not yet been properly interpreted (after 100 years of exciting, but not very productive debates).² I specifically appreciate the activity of Arcady Plotnitsky, philosopher studying Bohr’s views, see, e.g., [236]–[238]. He teaches us (participants of Växjö conferences) a lot. First of all we got to know that the Copenhagen interpretation cannot be rigidly coupled with Bohr’s views. On many occasions Niels Bohr emphasized that QM is not about physical processes in microworld, but about our measurements [37]: “Strictly speaking, the mathematical formalism of quantum mechanics and electrodynamics merely offers rules of calculation for the deduction of expectations pertaining to observations obtained under well-defined experimental conditions specified by classical physical concepts”. The basic postulate of the Copenhagen interpretation of QM – “the wave function describes the state of a quantum system” (i.e., a concrete system, not an ensemble) – cannot be assigned to Bohr. Then we learned (again from Plotnitsky) that Bohr’s views have been crucially changed a few times during his life. Thus, there can be found many different Bohr’s interpretations of QM. Bohr was definitely the father of the *operational interpretation* of QM. As was already pointed out, Bohr emphasized that the formalism of QM does not provide the intrinsic description of processes in microworld, it describes only results of

²“When I speak with somebody and get to know their interpretation, I understand immediately it is wrong. The main problem is that I do not know whether my own interpretation is right.” (Theo Nieuwenhuizen) This is the standard problem of participants of Växjö conferences.

measurements. Bohr also can be considered as one of fathers of the so-called *information interpretation* of QM: the QM-formalism describes information about micro systems extracted by means of macroscopic measurement devices. Heisenberg (and to some extent Schrödinger) shared this viewpoint. Nowadays the information interpretation of QM became very popular, see, e.g., [130], [75], [238]. I can mention Anton Zeilinger [274] and Christopher Fuchs [76]–[78] among the active promoters of this interpretation; we can also mention Mermin’s paper [217].

One of the most memorable debates, on *Bell’s inequality*, was ignited by Luigi Accardi, see e.g. [2] [5] for his views, and Richard Gill, see e.g. [81]. They put 1000 Euro for (Luigi) and against (Richard) a possibility to simulate EPR-correlations in purely classical local framework, later the sum reached 3000 euro; I, Slava Belavkin, and Inge Helland agreed to be in the jury. (Very soon we realized that it was a wrong decision.) Although the positions of both parties involved in this great debate have not changed much after ten years of discussions, nor the jury was able to make a well-grounded decision, this debate had an impact in the quantum community, see, for example, [132]. The main message of this debate was that Bell’s arguments were not so well justified as it was commonly believed. A part of Bell’s critique [30] against von Neumann’s no-go theorem can be redirected against Bell’s own theorem. And it is not as easy as it was believed to defend the Bell’s position.

As an expert in probability, in general, I agree with Luigi Accardi. (But I am not sure that computer simulation can play the role of a crucial argument.) This inequality is a general statistical test to check a possibility to fit the data collected in a few experiments into a single *Kolmogorov probability space*, see monograph [174] for the complete analysis of this problem, see also [119], [121], [122], [202], [210]–[213], [123], [125], [132], [129], [9]. This is a test of a possibility to use one special model of probability theory, the Kolmogorov model [194], for the collected data. I recall that already Lobachevsky and Gauss planned experiments to check applicability of the Euclidean model of geometry, see Section 4 for more detail. They proposed concrete tests. Bell’s inequality is a similar test for Kolmogorov’s probability model. We recall that, in fact, Bell’s inequality was invented many years ago by Boole [39], [40]: to check that statistical data can be described by a single Boolean algebra (see also I. Pitowsky [234] for a detailed analysis of the problem). General statistical tests of Kolmogorovness

of data were found by Soviet mathematician Vorob'ev [271]. Where does non-Kolmogorovness come from? It is a separate problem. Two possibilities were mentioned by Bell: violation of locality or (and) violation of realism. But, in principle, non-Kolmogorovness can come from various sources different from those mentioned by Bell, see my monograph [174]. Thus the honest position would be that violation of Bell's inequality can only tell us that the Kolmogorov's model did not pass the test. We cannot derive the definite conclusion on concrete sources of non-Kolmogorovness.

It is not easy to understand why physicists do not like such a viewpoint (with one exception: the famous Soviet experimenter in quantum optics Klyshko [191]). Theoretical physics is about creation of mathematical models of reality. Elaboration and experimental verification of the statistical test, the Bell-Boole inequality, which shows the boundary of applications of one of mathematical models, Kolmogorov's one, is the great success! If one does not like to proceed in such a way, then they should take other possible sources of non-Kolmogorovness not less seriously than Bell's sources. For example, my graduate student Guillaume Adenier studied the impact of the so-called *unfair sampling* [11]–[21], impossibility to guarantee reproducibility of statistical properties of ensembles used in experiments with incompatible pairs of orientations of polarization beam splitters.

It is clear that unfair sampling implies non-Kolmogorovness and, hence, violation of Bell's inequality. And from our viewpoint, unfair sampling is not a less important source of non-Kolmogorovness than, e.g., nonlocality. In turn there are various sources of unfair sampling. One of the most well known is the inefficiency of detectors. I spoke a few times with Alain Aspect about this problem. He was not even sure that experimenters should concentrate efforts to close this “loophole”. For him, this source of violation of Bell's inequality can not be considered on the same level of importance as “Bell's sources”. Opposite to him, I think that closing of the “efficiency of detectors loophole” is not less important than, e.g., locality loophole; without this it is totally meaningless to try to restrict sources of non-Kolmogorovness to “Bell's sources.” From the purely probabilistic viewpoint it is even more natural to expect that non-Kolmogorovness is induced by the cutoff of ensembles. Therefore, during a last few years, I and Adenier advertised the EPR-Bell experiment with *Tungsten-based Superconducting Transition-Edge Sensors* (W-TESs) – the ultra-sensitive

microcalorimeters, see, e.g., [176] for an experimental proposal.³ An interesting general test of fair sampling was elaborated in [14]. Unfortunately, we were not able to convince experimenters to do this test (in spite of a number of promises, it has never been done). I also point to the really important contribution of Jan-Ake Larsson [201] to the study of the probabilistic structure of the problem of the detectors efficiency.

Another source of unfair sampling is the use of the *time window* in the EPR-Bell experiments; it also makes cutoff of ensembles inducing non-Kolmogorovness, see [94]–[96], [81], [203], [61], [62], [167] for different viewpoints on this problem. Unfair sampling need not be reduced to ensemble cutoff, as due to inefficiency of detectors or time window. Unfair sampling can take place for 100%-efficient detectors and practically zero time window. It appears naturally in models of the EPR-Bell experiment taking into account parameters of measurement devices [174], [168]. In such models randomness induced by preparation procedure is combined with randomness induced by measurement device.

The majority of participants of Vaxjo conferences believed that QM, as it stands now, will, for a long time to come or even indefinitely, remain a correct, or indeed the correct, theory within its proper scope. On this view, improvements in our experimental technology would not alter the essential features of QM in its present form, although new exciting experimental and theoretical findings, including of foundational nature, are possible. Indeed, even in this group, there was no consensus not only on whether there is a single correct interpretation of QM but also on whether we really have a proper foundation of the theory, say, of the type we have in relativity theory. Several papers presented in Vaxjo explored this question and possible new foundational approaches to the standard version of QM.⁴

³Paul Kwiat tried to design such an experiment in his lab a few years ago. He promised me a talk on such an experiment at Vaxjo conferences 2006, 07, but, as I understood, his group did not overcome technical problems. Marco Genovese works on this problem right now; Anton Zeilinger recently, February 2010, told me that his group will soon start such experiments. It is clear that this problem (unfair sampling for the Bell’s tests with photons) cannot be solved with the threshold type detectors, photomultiplier tubes – PMTs, avalanche photodiodes – APDs, visible light counters – V LPCs.

⁴I can mention V. Belavkin, B. Coecke, W. De Muynck, C. Fuchs, R. Schack, M. Appleby, A. Plotnitsky, L. Hardy, M. D’Ariano, A. Elitzur, W. Zurek, P. Busch, D. Greenberger, R. Balian, K. Svozil, J. Smolin, M. Ozawa, A. Peres, D. Mermin, S. Stenholm, J.

On the other hand, a number of participants actively promoted alternative models, which might enable us to *go beyond the standard QM*. In particular, some of them considered QM as an emergent theory and envisioned, “dreamed of,” the possibility of a reduction of quantum randomness (viewed as irreducible by the orthodox interpretation) to classical randomness, for example, of the type found in classical statistical physics.⁵ The present author belongs to this (“minority”) group and considers *QM as an approximation, possibly a very good one, of a more fundamental theory of microscopic reality*.

I emphasize that the reduction of quantum randomness to classical randomness, e.g., consideration of quantum systems as classical random signals (as in the present book), does not imply a kind of comeback to Laplacian determinism. Consider the standard Brownian motion. Suppose that we are not able to take into account the effect of the random background for an individual particle. In such a situation we are not able to predict the trajectory of this particle, even if the initial conditions are determined with high precision. Nevertheless, the impossibility to predict the trajectory is not interpreted as the absence of the trajectory. (In [19] it was shown that classical Brownian motion exhibits even such a “fundamentally quantum property” as entanglement.)

More generally, it may be argued that strategically there are two main ways *to go beyond QM*. The first is to stimulate the development of the conventional QM, especially by designing new experiments, in a hope that, sooner or later, QM will reach the limit of its validity, even within its proper experimental scope (i.e., as a nonrelativistic theory of quantum phenomena), “peacefully,” as it were, as the result of its own, internal development. The second approach is to pursue a critique of the conventional approach in order to find its weak points, properly handling which would require an alternative theory.

I support both approaches; and, as I said, I believe, with Einstein, that the standard QM will ultimately prove to be an *approximation of a more fundamental theory*, perhaps based on a prequantum mathematical model of the type described above. At the moment, however, I do not think that any available model of this type provides a viable possibility in this regard. Mathematics offers great opportunities to

Summhammer, P. Lahti, I. Volovich.

⁵I can mention G. ‘t Hooft, T. Elze, C. Garola, M. Davidson, T. Boyer, T. Nieuwenhuizen, S. Gudder, G. Emch, B. Hiley, C. Wetterich, D. Cole, L. de la Pena, G. Adenier, H. D. Doebner, A. F. Kracklauer, Ch. Roychoudhuri.

explore, to “play with,” various pre-quantum models. However, such mathematical games are not the same as real physical models, which must rigorously relate their mathematics to experimentally observed phenomena. The main problem of prequantum approaches, including my own, is the lack of proposals for realistic experiments that could demonstrate that the QM-formalism describes micro-reality only by a way of approximations⁶ that can be superseded by a better theory. Nevertheless, these approaches offer a possible new trajectory for future development of quantum theory, and the main aim of Växjö conferences was to explore such trajectories.

Overall, Växjö conferences have played an important role in the ongoing investigation of quantum foundations and possibilities of new discoveries in QM and beyond it⁷, for example, in quantum field theory. Several additional aspects and high points of the conferences are worth mentioning here. The *interaction of a large number of experimenters* at the conferences (e.g., A. Aspect, H. Rauch, G. Weihs, M. Genovese, S. Kulik, C. Roychoudhuri, M. Zukowski, F. De Martini, A. Zeilinger, F. Sciarrino, B. C. Hiesmayr, C. Roos,...) will, hopefully, facilitate the aim of designing new foundational experiments, especially but other tests of possible violations of Bell’s inequalities, the approach that dominated this field for a long time. The discussions and debates on the foundations of quantum information theory (especially quantum cryptography and computing) that took place during the conference will, undoubtedly, contribute to further developments in this important field of research. Indeed, quantum information theory can be considered as a great experiment to test the validity of the main principles of quantum mechanics and its interpretation. Finally, Växjö conferences offered significant new insights into the question of quantum probability, which remains essential to quantum foundations, however one pursues them.

1.2 The role of probability

Main message: *Probability in QM should be taken seriously; this is a tricky notion; even “classical probability” can induce rather counter-*

⁶My classical field-type model predicts that even the basic rule of QM, Born’s rule, is an approximate rule.

⁷See, e.g., [1]–[6], [17]–[18], [20], [21], [23], [24], [27]–[29], [33], [44], [45]–[50], [52], [57]–[59], [75]–[78], [81], [88], [91], [93]–[99], [106], [137] [131], [132], [158], [190] [202], [202], [206], [217], [224], [233], [236], [245], [257], [259], [266], [272].

intuitive (paradoxical) conclusions. Therefore a rigorous mathematical presentation of probabilistic statements of QM is very important.

In fact, *quantum probability* or, to be more precise, the difference between quantum and classical probabilistic models was the starting point of my interest in quantum foundations. Graduated from the Department of Mechanics and Mathematics of Moscow State University I was well trained in probability theory; on a few occasions I was lucky to meet the founder of the modern probability theory Andrei Nikolaevich Kolmogorov [194]. In particular, he communicated to Doklady USSR paper [254] (written under supervision of Oleg Georgievich Smolyanov) devoted to a generalized model with complex valued probabilities. My PhD-thesis [113] was devoted to the theory of *probability on infinite-dimensional spaces* (including distributions on Hilbert spaces) with applications to mathematical problems of quantum field theory [114]–[117].

This pathway to quantum theory, namely, through theory of continual integral, secured me for a long time from terrible problems and prejudices related to quantum foundations which everybody meets immediately in standard textbooks. The continual integral approach, especially its Euclidean version, induced an illusion that quantum theory, at least quantum field theory, is about integration on infinite-dimensional spaces. During quite long time I was completely sure that the infinite number of the state space dimensions is the main point of departure of QM from classical mechanics.

The main surprise for me in von Neumann's book [270] was the notion of *irreducible quantum randomness*. Von Neumann sharply distinguished classical and quantum randomness. The first one, which is exhibited everywhere, besides the QM (in classical statistical mechanics, economics, finances, biology, engineering), is a consequence of the impossibility to take into account all parameters describing a system and its interaction with a measurement device (or environment). Although it is difficult and sometimes even really impossible to specify the values of these parameters (so to say "hidden variables"), there are no doubts of their existence. For example, in statistical mechanics the Liouville equation describes dynamics of the probability distribution on a phase space. It is very difficult to solve the corresponding system of Hamiltonian equations describing trajectories of millions of individual particles. Nevertheless, there are no doubts that these particles really move in physical space. The easiest way to describe diffusion (including the Brownian motion) is to solve the Fokker-Planck (and,

in general, the direct Kolmogorov) equation for the probability distribution on configuration space. Still, it is also possible to solve the corresponding stochastic differential equation and obtain the description of dynamics by means of trajectories of individual particles, the classical stochastic process.

In contrast, it has been claimed that quantum randomness cannot be reduced to our lack of knowledge of “hidden variables”. It became rather fashionable to claim that QM has demonstrated the violation of laws of classical probability theory, see, e.g., Richard Feynman [74], p. 2: “But far more fundamental was the discovery that in nature the laws of combining probabilities are *not* those of the classical probability theory of Laplace.”

Such a viewpoint supports the illusion that something mystical goes on in QM, since the probability laws which has been valid everywhere are violated in the microworld. In my first studies I clarified this problem⁸. One cannot speak about probability without specifying a mathematical model of probability. In particular, the notion of “classical probability” is related to a variety of mathematical models: Kolmogorov’s measure-theoretic, von Mises’ frequency, subjective probability and so on [267]–[269], [121]. Quantum probability is nothing else than a probability described by Dirac-von Neumann model [63], [270]: a complex Hilbert space; wave functions and, more generally, density matrices as states; self-adjoint operators as observables; Born’s rule. Therefore, when one speaks about matching or mismatching of classical and quantum probability, the classical model and, what is very important, *matching rules should be specified*.

At the first stage of my exciting journey into the probabilistic foundations of QM, I found that there is no contradiction between the QM probabilistic model and von Mises’ frequency probability theory [121], [123], [126], [128], [129], [126]. And this is not surprising. The von Mises approach [267]–[269], [121] is a very general empiric approach providing the probabilistic model for statistical data collected in experiments. In fact, von Mises (in 1919) elaborated the probabilistic model based on the empiricist ideology: statistics of outcomes of experiments is described by frequencies of realizations. Thus, *the classical probabilistic model of R. von Mises does not contradict QM*. This was well known already to von Neumann, who proposed to consider

⁸See [267]–[269], [121], [123], [126], [128], [129], [126], [128], [119], [121], [125], [134], [135], [137]–[139], [142], [145], [147], [174].

von Mises theory as the probabilistic ground of QM.⁹ The approach of von Mises is especially useful to describe statistical data collected in quantum experiments. It reflects the temporal structure of an experimental data-stream. And a typical quantum experiment has the well defined temporal structure. This is a run of preparations and corresponding detections.

The main difference between von Mises' and von Neumann' views was that von Mises did not claim that additional variables responsible for randomness could not be introduced.

1.3 To Hilbert space from probability

Main message: *The Hilbert space model provides a linear space representation of probabilistic data.*

In his famous book [208] Mackey started in purely empiricist manner, i.e., with probabilities collected in various (in general incompatible) experiments, and tried to find conditions inducing representation of data by complex probability amplitudes, or in the abstract framework by normalized vectors in the complex Hilbert space.¹⁰ Unfortunately, he did not succeed completely; finally, the complex Hilbert space representation was postulated and this decreased the value of Mackey's studies. I spent a few years by working to complete Mackey's program and I developed the general contextual approach based on families of probabilities obtained by measuring of various observables for various contexts. I found an algorithm representing probabilities (under special, but natural conditions) by complex amplitudes – *quantum-like representation algorithm* (QLRA); probabilities and amplitudes are coupled by Born's rule, i.e., the squared amplitude coincides with the probability. Thus, the complex Hilbert space was not

⁹“However, the investigation of the physical quantities related to a single object S is not the only thing which can be done – especially if doubts exist relative to the simultaneous measurability of several quantities. In such cases it is also possible to observe great statistical ensembles which consist of many systems S_1, \dots, S_N (i.e., N models of S , N large). (Such ensembles, called collectives, are in general necessary for establishing probability theory as the theory of frequencies. They were introduced by R. von Mises, who discovered their meaning for probability theory, and who built up a complete theory on this foundation.)” See [270], p. 298.

¹⁰I called this problem the “inverse Born problem” [159], [174], [226]. Born's rule obtains probability from the wave function. We are interested in production of a complex amplitude from probabilistic data. This amplitude has to satisfy to the “direct Born's rule.”

postulated (as was done by everybody, from Dirac [63] and von Neumann [270] to Mackey [208]), but appeared in the constructive way [174].¹¹

1.4 Against completeness

Main message: *In the past, various models of reality have been claimed to be final (complete). The most known examples are Euclidean geometry (see, especially Kant [109]) and Newtonian mechanics. However, sooner or later such scientific myths died. QM is the latest myth of a complete theory.*

My next action towards demystification of the QM-formalism was against Bohr's thesis on *completeness of QM*. I remark that von Neumann's statement of irreducibility of quantum randomness is nothing else than the probabilistic performance of Bohr's statement of *completeness of QM*. By Bohr QM provides the finest possible description of micro phenomena; a finer description of the state of a quantum system than given by the wave function is totally impossible. To justify completeness of QM, Bohr need not any "no-go theorem" (such as von Neumann's, Kochen-Specker's or Bell's theorems [270], [193], [30]). He was completely fine with Heisenberg's uncertainty relation.

Einstein had never accepted Bohr's thesis on completeness of QM. All his life he dreamed of creation of a new fundamental theory of micro phenomena. He was sure that the wave function does not provide the complete description of the state of an individual quantum system. Einstein was the father of the *ensemble interpretation* of the wave function as describing statistical properties of an ensemble of systems created by some preparation procedure.¹² Part of the quantum community borrowed Einstein's ensemble interpretation of the

¹¹Moreover, it was found that the formalism of QM is too restrictive for a general empiricist model: not all probabilities are represented by complex amplitudes. Some of them are represented by the so-called hyperbolic amplitudes [136], [138], [143], [174], [188] (taking values of the form $z = x + jy, x, y \in \mathbf{R}, j^2 = +1$) or mixed hyper-complex amplitudes. Thus the complex Hilbert space is too restrictive for the linear representation of the general empiricist (contextual) model; the hyper-complex Hilbert space provides the proper base.

¹²This interpretation was later elaborated by Leslie Ballentine [25], [26] who used the term *statistical interpretation*. Unfortunately, this terminology is rather misleading, since it had been used by von Neumann, too: the wave function, although assigned to the state of an individual system, expresses statistics of measurements (but this statistics is coupled to irreducible randomness).

wave function and combined it peacefully with Bohr's thesis of completeness of QM. They support the *operational or empiricist interpretation* of QM [207], [43], [229], [230], [102], [103], [192], [51], [49] and consider QM formalism as a story on preparation and measurement procedures. At this stage the positions of Bohr and Einstein coincide. However, Einstein did not see any fundamental barrier to complete QM (to know what goes on behind rough macroscopic preparation and measurement procedures. Majority of people using the operational interpretation do not support Einstein's views. In contrast to Einstein, they do not dream of new, more sensitive preparations and measurements which will show that QM provides only an approximative description of phenomena. But even this interpretation is better than the orthodox Copenhagen interpretation, according to which the wave function provides the complete description of the state of a micro-system.

Thus *the first dream of Albert Einstein was to reduce quantum randomness to classical randomness by creating a finer description of micro-system's state than given by the wave function*. It was the dream of creation of a kind of classical statistical mechanics for microsystems. Einstein did not specify prequantum, so to say hidden, variables. They need not be the canonical pair of (q, p) , position and momentum. Classical prequantum statistical mechanics has to be based on some sort of deterministic dynamics. For example, one may expect to find a micro-analog of the Liouville equation and the underlying Hamilton equations. We emphasize that Einstein, whose contribution to theory of Brownian motion is well known, did not dream of the comeback to the Laplacian determinism. (An essential part of book [68] was devoted to critique of mechanical determinism.) The prequantum deterministic dynamics is definitely influenced by classical randomness of the initial conditions and random background.

1.5 Einstein's dream of the pure field model

Main message: *Particles are illusions and fields are reality.*

It is less known that Einsteinian "beyond quantum world" was not imagined as a micro-copy of our macroscopic world populated by particles. Einstein's greatest dream was to *eliminate particles totally from coming fundamental theory*. New theory should be a purely field model of reality: no particles, but only fields (or may be just one field). He considered particles as the relict of the old mechanistic

model of reality. It may be not so well known, but for Einstein QM was not a theory too novel (so that he even could not understand it, as some people claim), but, in contrast, it was too old fashioned to be considered a new fundamental theory.

In [68] he discussed a lot Bohr's principle of complementarity, so to say, wave-particle duality. He was not happy with the quantum jargon mixing waves and particles. Einstein was sure that the wave-particle duality will be finally resolved in favor of a purely wave model:

“But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics? What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created. Its final aim would be the explanation of all events in nature by structure laws valid always and everywhere. A thrown stone is, from this point of view, a changing field, where the states of greatest field intensity travel through space with the velocity of the stone. There would be no place, in our new physics, for both field and matter, field being the only reality. This new view is suggested by the great achievements of field physics, by our success in expressing the laws of electricity, magnetism, gravitation in the form of structure laws, and finally by the equivalence of mass and energy. Our ultimate problem would be to modify our field laws in such a way that they would not brake down for regions in which the energy is enormously concentrated. But we have no so far succeeded in fulfilling this program convincingly and consistently. The decision, as to whether it is possible to carry it out, belongs to the future. At present we must still assume in all our actual theoretical constructions two realities: field and matter.”, see the book of Einstein and Infeld [68], p. 242-243. Then they discussed QM and a possibility to interpret the wave function, the probability wave, as a physical field:

“For one elementary particle, electron or photon, we have probability waves in a three-dimensional continuum, characterizing the statistical behavior of the system if the experiments are often repeated. But what about the case of not one but two interacting particles, for instance, two electrons, electron and photon, or electron and nucleus? We cannot treat them separately and describe each of them through a probability wave in three dimensions, just because of their

mutual interaction. Indeed, it is not very difficult to guess how to describe in quantum physics a system composed of two interacting particles. We have to descend one floor, to return for a moment to classical physics. The position of two material points in space, at any moment, is characterized by six numbers, three for each of the points. All possible positions of the two material points form a six-dimensional continuum and not a three-dimensional one as in the case of one point. If we now again ascend one floor, to quantum physics, we shall have probability waves in a six-dimensional continuum and not in a three-dimensional continuum as in the case of one particle. Similarly, for three, four, and more particles the probability waves will be functions in a continuum of nine, twelve, and more dimensions. This shows clearly that the probability waves are more abstract than the electromagnetic and gravitational field existing and spreading in our three-dimensional space,” [68], p. 290-291.

This discussion is very important for our further studies of a possibility of creation of purely wave picture of physical reality. It was directly emphasized that one of the main problem is the impossibility to realize quantum “waves of probability” for composite quantum systems on physical space. Recently this problem was solved by the author of this book, see [171], [172], [179]–[181], [184]–[187], and the solution will be presented in Chapter 2, Section ?? . Finally, Einstein and Infeld concluded, [68], p. 293:

“But there is also no doubt that quantum physics must still be based on the two concepts: matter and field. It is, in this sense, a dualistic theory and does not bring our old problem of reducing everything to the field concept even one step nearer realization.”

As we have seen, Einstein’s picture of electron or neutron is very simple: these are fields densely concentrated in small areas of space. These are classical fields (not quantum!). Hence, Einstein was sure that the classical space-time picture of reality could be combined with QM. (We state again that classical has the meaning classical field theory and not at all classical mechanics of particles).

1.6 Anti-photon

Main message: *Photon is a pulse of classical electromagnetic field. Discreteness is an illusion produced by detectors.*

Now we discuss the notion of photon. It is well known that Albert

Einstein invented the notion of the *quantum of light* which was later called photon.¹³

Bohr was not happy with the invention of light quanta. In particular, the *Bohr-Kramers-Slater theory* [38] was an attempt to describe the interaction of matter and electromagnetic radiation without using the notion of photon. We also mention the strong opposition to the notion of photon from two fathers of QM: Alfred Lande [199], [200] (in particular, this name is associated with Lande g -factor and the first explanation for the anomalous Zeeman effect) and Willis E. Lamb [198] (e.g., Lamb shift). Their views on electromagnetism differ crucially from the view of Albert Einstein (at least, young Einstein, see Section 1.1.9 for the evolution of the Einstein views). The latter wrote in 1910 [67], p. 207: “What we understand by the theory of “light quanta” may be formulated in the following fashion: a radiation of frequency ν can be emitted or absorbed only in a well defined quantum of magnitude $h\nu$. The theoreticians have not yet even come to an agreement in regard to the following question: Can the light quanta be accounted for entirely by a characteristic of the emitting or absorbing substance, or should the electromagnetic radiation itself be assigned, besides a wave structure, such that the energy of the radiation itself is already divided in definite quanta? I believe that I have proven that this latter view should be adopted.”

Both Lande and Lamb rejected the existence of discrete quanta of electromagnetic field. They were sure that one can proceed in the so-called *semiclassical approach*, describing the interaction of classical electromagnetic field with quantum matter, see, e.g., [249], [209], [205] and recently [195], [196], [242], [243]. We cite Lamb [198], p. 211: “It is high time to give up the use of the word “photon”, and of a bad concept which will shortly be a century old. Radiation does not consist of particles...” For adherents of the semi-classical approach quantization of the electromagnetic field is done by detectors; it is not present in electromagnetic field propagating in the vacuum.¹⁴ We also

¹³Originally the concept of photon was invented by physical chemist G. N. Lewis who really considered photons as light particles that transmit radiation from one atom to another. Wave-like properties of photon were attributed to guiding ghost field. See Lamb’s “Anti-photon” [198], p. 201-211, for more details. We underscore the difference, the photon-terminology, unlike the quantum of light terminology, is not so innocent as one may think. By calling the quantum of light the photon people emphasized the role of a particle picture of light.

¹⁴The semiclassical approach can describe a number of quantum effects, e.g., the pho-

recall that Max Planck opposed Einstein's idea of quantum of light from the beginning, and remained a champion of the unquantized Maxwell field throughout his life. In 1907 in a letter to Einstein, he said: "I am not seeking the meaning of the quantum of action (light quantum) in the vacuum but rather in places where emission and absorption occur, and I assume that what happens in the vacuum is rigorously described by Maxwell's equations," see, for example, [222].

We also mention *stochastic electrodynamics* (SED) – a variant of classical electrodynamics which postulates the existence of a classical Lorentz-invariant radiation field (zero point field, Marshall and Bradford, Boyer, de la Pena and Cetto, Coli see, e.g., [41], [53]–[57], [46], see also [42], [224]). The presence of this field plays the crucial role in SED's description of quantum effects. We recall that already in 1911 Planck introduced the hypothesis of the *zero point electromagnetic field* in an effort to avoid Einstein's ideas about discontinuity in the emission and absorption processes.

It is important for our further considerations, that neither the semiclassical approach, nor SED, resolve the wave-particle dualism. Neither was it resolved by Bohmian mechanics, the modern version of De Broglie's double solution approach. Bohmian mechanics reduces the quantum randomness to classical ensemble randomness, and particles are the basic objects of this theory.

1.7 Schrödinger's wave mechanics

Main message: *Wave mechanics is an alternative to the Laplacian deterministic model of particles' motion*

By now, the reader might be wondering that Schrödinger's name has not yet been mentioned in the discussion of classical and quantum wave mechanics. I refrained from it till this chapter to have enough place to consider not only Schrödinger's wave mechanics, but also his philosophic doctrine, that played an important role in my own theory.

At the beginning Schrödinger considered the squared wave function of the electron (multiplied by its electric charge e) as the density of its charge:

$$p(t, x) = -e|\psi(t, x)|^2.$$

toelectric effect (G. Wentzel and G. Beck, 1926; see W. E. Lamb and M. O. Scully [197] for more detailed calculations).

The solution $\psi(t, x)$ of Schrödinger's equation for, e.g., hydrogen atom describes oscillations of such electronic cloud, which induce electromagnetic radiation with frequencies and intensities matching the experiment.¹⁵ This picture of a quantum particle as a field, in this case electronic field, coincides with the picture from Einstein's field dream. Unfortunately, Schrödinger was not able to proceed in this way. He understood, as well as Einstein, that already for two electrons the wave function cannot be interpreted as a field on a physical space: $\psi(t, x, y)(x = (x_1, x_2, x_3), y = (y_1, y_2, y_3))$, is defined on \mathbf{R}^6 . Although formally Schrödinger gave up and accepted Born's interpretation of the wave function, he did not like the Copenhagen interpretation, as Einstein did neither, especially, Bohr's thesis on completeness of QM. Schrödinger dreamed to go beyond QM, and to refind purely wave resolution of wave-particle duality. Nor, however, did Schrödinger accept Einstein's ensemble interpretation of the wave function. No wonder why! Schrödinger would rather have a wave associated with an individual quantum system, and the wave function was the best candidate for such wave. Therefore, he rejected Einstein's idea to associate the wave function with an ensemble of quantum systems, see their correspondence in [73].

As I mentioned, Einstein did not want the comeback to the Laplacian determinism, and Schrödinger did neither: Schrödinger's views on scientific description of physical reality were based on a well elaborated approach, the so-called *Bild-conception tradition*, see D'Agostino [47], p. 351, for details:

Schrödinger called "the classical ideal of uninterrupted continuous description", at both observables' and theoretical levels, an "old way", meaning, of course, that this ideal is no longer attainable. He acknowledged that this problem was at the center of the scientific debate in the Nineteenth and Twentieth centuries as well:

"Very similar declarations...(were) made again and again by competent physicists a long time ago, all through the Nineteenth Century and the early days of our century...they were aware that the desire for having a clear picture necessarily led one to encumber it with unwarranted details," [34], p.24.

I would like now to cite a rather long passage from D'Agostino [47], pp. 351-352, presenting philosophic views of Schrödinger on two levels

¹⁵Unfortunately, I was not able to find in Schrödinger's papers any explanation of the impossibility to divide this cloud into a few smaller clouds, i.e., no attempt to explain the fundamental discreteness of the electric charge.

of description of reality: observational (empiricist) and theoretical.

“The competent physicists are almost certainly Hertz, Boltzmann and their followers. One can thus argue that Schrödinger’s two-level conception above is, at bottom and despite its “amazing” appearance, part of the tradition of the nineteenth-century Bild-conception of physics, formulated by Hertz in his 1894 *Prinzipien der Mechanik*, and also discussed by Boltzmann, Einstein et al. He partially inherited this tradition from his teacher Exner and he deepened this conception through his intense study of Boltzmann’s work. One of the main features of the above tradition is its strong anti-inductionism. If theory is not observation-dependent - in the sense that it is not constructed on (or starting from) observations - it consequently possesses a sort of distinction as regards observations. This distinction may be pushed to various degrees of independence. Hertz implied that a term-to-term correspondence between concepts and observables was not needed when he introduced hidden quantities among the theory’s visible ones. In his often quoted dictum, Boltzmann asserted that only one half of our experience is ever experience. At bottom, Schrödinger was thus orthodox in his assertions that theory and observations are not necessarily related in a term-to-term correspondence and a certain degree of independence exists between them. However, when he added the further qualification that a repugnancy might exist between them, he stretched this independence to its extreme consequences, introducing a quasidichotomy between a pure theory and an observational language.

This extreme position was not acceptable to the majority of his contemporaries and to Einstein in particular. Causal gaps, even if limited to the observables level, could not be accepted by Einstein and other scientists. In fact, Einstein’s completeness implied a *bi-univocal* correspondence between concepts and observables. It followed from Einstein’s premises that, if Schrödinger’s wave function did not correspond to a complete description of the system, the reason was to be sought in its statistical (in Einstein’s sense!) features: i.e. Schrödinger’s wave function refers to an ensemble not to an individual system. Differently, Schrödinger thought that incompleteness in description was generated by an illegitimate (due to indistinguishability) individualization of classical or quasi-classical particles in microphysics. On the other hand, Schrödinger could not accept Heisenberg’s and Bohr’s Copenhagenism, because, for him, their position represented a concession to an old conception of the theory-

observations relation, implying that causality-gaps and discontinuities on the observation-level would forbid the construction of a complete theory (a complete model). One can thus argue that Schrödinger considered the fundamental defect of the Copenhagen view its missing the distinction between the two levels of language, the descriptive and the purely theoretical level. From the QM impossibility of a continuous descriptive language on the observable level, the Copenhagenists would have rushed to conclude the uselessness of a continuous purely theoretical language.”

In this book I present a theoretical (causal and continuous) model of physical reality, *prequantum classical statistical field theory* – PCSFT.¹⁶ Since my starting point was not the observations, my model does not rely completely on the descriptive language of QM, which fact is in total accordance with views of Boltzmann, Hertz, Exner, and Schrödinger on relation between theoretical and observational models. The correspondence between concepts of PCSFT and QM is not straightforward, see Chapter ?? for coupling of PCSFT and QM through a measurement theory for PCSFT.

Let us return to the views of Schrödinger on QM and physical reality. I cite from Lockwood [204], pp. 385-386:

“Two possibilities then present themselves. One possibility (a) is that individual physical systems do, after all, possess determinate states in essentially the classical sense. That is to say, the classical dynamical variables do have well-defined values at every moment, arbitrary precise simultaneous knowledge of which is, however, in principle unattainable. Consequently, we have to fall back on statistical statements. The assertions of quantum mechanics should accordingly be understood to refer, as in statistical mechanics, to the distribution of values of these variables within an ideal ensemble of similarly prepared systems. Schrödinger assumed this to be Einstein’s position. The other possibility (b) is that the quantum-mechanical description, as embodied in the ψ -function, is a complete specification of an objectively “fuzzy” state. On this conception, quantum mechanics does offer a model of reality; but the model it presents us with is of an objectively “blurred” reality. The difference between these two interpretations, Schrödinger regards as analogous to the difference between

¹⁶In principle, causality is approachable in the PCFT-framework, but the situation is quite complicated, because of the presence of vacuum fluctuations; we shall come back to this problem in Section 2.7

an out-of-focus photograph of something with perfectly sharp outlines, and a properly focused photograph of something lacking sharp outlines, such as a patch of fog. Having set up these alternatives, Schrödinger then, disconcertingly, proceeds to argue that neither is tenable.”

In fact, the viewpoint to QM generated by PCSFT in combination with the corresponding threshold detection model (TSD) does not match neither with (a), the classical statistical mechanical viewpoint to QM, nor with (b), the completeness viewpoint to QM. The aforementioned two level description of reality based on the combination of theoretical and observational models is sufficiently close to the one given by PCSFT/TSD. However, opposite to Schrödinger and other adherents of the Bild concept, I think that all basic features of observational model have to be derivable from the theoretical model. This is a good place to point to a general scientific methodology which was advertized during many years by Atmanspacher and Primas [23]. Any scientific theory is based on two levels of description of reality: ontic (reality as it is) and epistemic (the image of reality obtained with the aid of a special class of observables). The QM-formalism is an example of an epistemic model. In this framework PCSFT can be considered as the ontic model and TSD as the epistemic model which is equivalent to QM. However, I am not fine with the notion of the ontic model as a model of objective reality, i.e., reality existing independently of our observational abilities. I rather prefer a two level description with two collections of variables, so to say fine and coarse variables. The coarse-variables are already approachable and the fine ones are yet not, but they will be in future. The coarse variables are determined by the fine ones. However, in PCSFT/TSD approach this realition is very tricky, see Section ?? for further discussion.

1.8 Bohr-Kramers-Slater theory

In paper [38] Bohr, Kramers, and Slater (BKS) tried to treat the interaction of matter and electromagnetic radiation without photons. By their model atoms produce a virtual field (induced by virtual oscillators) which induces the emission and absorbsion processes. This virtual field contains contributions of all atoms and hence each transition in a single atom is determined by processes in all atoms nearby. The BKS-theory can be coupled with PCSFT. In the latter any “quantum particle” is represented by a classical random field. In particular,

any atom is nothing else than an atomic field. A group of atoms induces a collective atomic field. Therefore we might try to interpret the virtual BKS-field as the real atomic field of PCSFT. Any transition in atom (by the QM-terminology from one level to another) is a completely causal process of evolution of this field. Fields of various types of “quantum particles” can interact with each other or better to say there is a single fundamental prequantum field which have various configurations: photonic (electromagnetic), electronic, atomic,... In PCSFT we stress a role of the background field, vacuum fluctuations. This field is present even in the absence of “quantum particles”. The presence of the background field may solve one of the main problems of the BKS-theory, namely, possible violation of the law of conservation of energy on the individual level: the impossibility to account for conservation of energy in a process of de-excitation of an atom followed by excitation of a neighboring one. In PCSFT the energy of the fundamental prequantum field is not changed in this process.

The BKS-theory was an attempt to unify wave and particle pictures on the basis of the classical field theory. This was an attempt of *causal continuous description* of quantum jumps in the processes of absorption and emission. We remark that Bohr elaborated his principle of complementarity only because he was not able to construct a satisfactory causal field-type model. Later he advertised completeness of QM [36], [37]. Roughly speaking he tried to stop studies similar to his own in 1924th. (The Freudian background of such behavior is evident.)

1.9 On the evolution of Einstein’s views: from classical electrodynamics to photon and back

Einstein has views, as presented respectively in Sections 1.5 and 1.6, appear to be in conflict. On the one hand, as discussed in Section 1.5, he was the discoverer of the particle of light, the photon, as it eventually became known, and thus advocated the particle-like model of the behavior of light in certain circumstances, a view confirmed by the Compton scattering experiment in 1923, shortly before the discovery of quantum mechanics. On the other hand, as discussed in Section 1.6, he championed the classical-like field model as the best, if not the only model, for fundamental physics, which, given the continuous character of the classical field theory, is difficult to reconcile with the concept of photon. This discrepancy leads one to suspect that these

two positions reflect the views of two different “Einsteins,” especially given that they correspond to two different periods of Einsteins work, the first, roughly between 1905-1920, and second, from roughly 1920 to his death in 1955. His book with Infeld, discussed in 1.6, was originally published in 1938 and, thus, it might be added, was written shortly after the EPR argument, which solidified Einstein’s critical assessment of quantum mechanics. The evolution of Einstein’s views is instructive and one might sketch this evolution roughly as follows.¹⁷

It may be argued that Einstein’s primary model for doing fundamental physics was had always been Maxwell’s electrodynamics as a field theory, which grounds special relativity, introduced in 1905, the same year he introduced the idea of photon. It also appears, however, that his thinking at the time was more flexible as concerns what type of physical theory one should or should not use. His approach was determined more by the nature of the experimental phenomena with which he was concerned, or in his own later words, his attitude was more “opportunistic” [246], p. 684, rather than guided by a given set of philosophical preferences, as in his later works. In this respect, the term “opportunistic” may no longer easily apply to his later thinking, or at least his opportunism was conditioned by his philosophical inclinations toward a classical-like field-theoretical approach to fundamental physics. Einstein appears to have introduced the concept of photon under the pressure of experimental evidence, such as that reflected in Planck’s law or the law of photoeffect (for which Einstein was actually awarded his Nobel Prize). He went further than Planck by proposing that the photon was a real particle (rather than a mathematical convenience), the idea that took a while, until 1920s and much additional experimental evidence, most especially, again, Compton’s scattering experiments, to accept. Intriguingly, not only Planck but also Bohr was among the skeptics, and Bohr only accepted the idea in view of these experiments. Planck, who, as discussed earlier, strongly resisted Einstein’s introduction of the concept of the photon, had never reconciled himself to the idea. Thus, it appears that until roughly 1920, Einstein did not have a strongly held philosophical position of the type he developed later on, first, following

¹⁷The account of this evolution sketched here is courtesy of Arkady Plotnitsky [private communication]. See also Pais [232] for a discussion of the development of Einstein’s views on fundamental physics, from his earlier work to his work on general relativity and beyond; and for Einstein’s earlier views, see Don Howard and John Stachel, [105] and also [67]. For Einsteins later views, see especially both of his contributions to the Schilpp volume [246].

his work on general relativity (a classical-like field theory) and, secondly and most especially, in the wake of quantum mechanics. It is worth noting in this connection that he initially resisted Minkowsky's concept of spacetime as insufficiently physical, but eventually came to appreciate its significance, again, especially in view of its effectiveness in general relativity. It is true, however, that theoretical physics at the time, including quantum theory (the "old" quantum theory), was still more classically oriented, as against quantum mechanics in the Heisenbergian approach. In addition, given that, in some circumstances, light would still exhibit wave behavior, Einstein also believed at the time (until even 1916 or so), that a kind of new synthesis of the particle-like and the wave-like theory of radiation would be necessary. However, this hope had not materialized in any form that he found acceptable, and he was especially dissatisfied with Heisenberg's approach [92], developed into the matrix mechanics by Born and Jordan, or related schemes, such as Dirac's one [63]. The success of general relativity as a classical-like field theory was significantly responsible for strengthening Einstein's field-theoretical predilections, and shaped his program of the unified field theory (with a unification of gravity and electromagnetism as the first task), which he pursued for the rest of his life. The problems of quantum mechanics and his debate on the subject with Bohr continued to preoccupy him as well, as reflected in particular in his persistent thinking concerning the EPR experiment, on which he commented virtually until his death. His view of fundamental physics following his work on relativity was also more mathematically oriented than the earlier one. In particular, he came to believe that it is a free mathematical conceptual construction, such as those of Riemann's geometry and tensor calculus in the case of general relativity and indeed of a similar classical-like field-theoretical type, that should and, he even argued, will allow us to come closest to capturing, in a realist manner, the ultimate character of physical reality. He expressly juxtaposed this approach to that of the Copenhagen-Göttingen approach in quantum mechanics [246], pp. 83-85. In sum, Einstein had come to be convinced that a strictly field-like theory unifying the fundamental forces of nature should be pursued. He saw this kind of theory as the best and even, to him, the only truly acceptable program for the ultimate theory of nature, while he believed quantum mechanics to be a provisional theory, eventually to be replaced by a field theory of the type he envisioned.

It may be remarked that the idea of particle poses difficulties for

this view, especially the particle nature of radiation, initially represented in the idea of photon. This is why Einstein preferred and saw as more promising (than matrix mechanics) Schrödinger's wave mechanics, or why earlier he liked de Broglie's approach (which he used in his work on the Bose-Einstein theory). It is true that the latter does retain the concept of particle and, as such, represents an attempt at a synthesis of the wave and the particle pictures, which, as noted above, Einstein contemplated initially. Later on, however, he did not like Bohmian mechanics, which pursued a similar line of thinking, although his negative attitude appears to have been determined by a complex set of factors. Eventually it became apparent that Schrödinger's formalism could not quite be brought under the umbrella of Einstein's unified field-theoretical program, à la Maxwell, although in his later years (in 1940s-1950s) Schrödinger returned to his initial ideas concerning wave mechanics. Quantum electrodynamics and then other quantum field theories appeared even more difficult to reconcile with this approach. Even general relativity posed certain significant problems for Einstein's vision, such as singularities, eventually leading to ideas such as black holes, although the full measure of these difficulties became apparent only later on, after Einstein's death.

There is thus quite a bit of irony to this history. While Einstein was fundamentally responsible for several theoretical ideas that eventually led others to quantum mechanics, he had developed grave doubts about quantum mechanics as a "useful point of departure for future development" [246], p. 83. Since, however, our fundamental physics remained incomplete at the time, Einstein thought that his vision might ultimately be justified. It might yet be, since our fundamental physics still remains incomplete, and in particular, is defined by a manifest conflict between relativity and quantum mechanics or higher-level quantum theories. It would be curious to contemplate whether Einstein would have liked something like the string and brane theories, or any other currently advanced programs for fundamental physics and cosmology.

2 Prequantum classical statistical field theory: introduction

Now I turn to my model, PCSFT, which is based on the unification of two Einstein's dreams: to reduce quantum randomness to classical randomness and to create a purely wave model of physical reality. I emphasize from the very beginning that the majority of PCSFT-structures are already present in QM, but in PCSFT they obtain a new (classical signal) interpretation. Therefore the introduction in PCSFT presented in this section can be considered as a short dictionary that establishes a correspondence between terms of QM and PCSFT. However, PCSFT not only reproduces QM, but provides a possibility to go beyond it. Therefore, advanced structures of PCSFT do not have counterparts in QM.

2.1 Classical fields as hidden variables

Main message: *Quantum randomness is reducible to randomness of classical fields.*

Classical fields are selected as the hidden variables.¹⁸ Mathematically, they are functions $\phi : \mathbf{R}^3 \rightarrow \mathbf{C}$ (or, more generally, $\rightarrow \mathbf{C}^k$) which are square-integrable, i.e., elements of the L_2 -space. The latter condition is standard in the classical signal theory.

In particular, for electromagnetic field, this is just the condition of the finiteness of energy

$$\int_{\mathbf{R}^3} (E^2(x) + B^2(x)) dx = \int_{\mathbf{R}^3} |\phi(x)|^2 < \infty, \quad (1)$$

where

$$\phi(x) = E(x) + iB(x) \quad (2)$$

is the *Riemann-Silbertstein vector* (the complex representation of the electromagnetic field).

Thus, the state space of our prequantum model is $H = L_2(\mathbf{R}^3)$. Formally, the same space is used in QM, but we couple it with the classical signal theory. For example, the quantum wave function satisfies

¹⁸PCSFT is not a deterministic-type model with hidden variables. By fixing a classical “prequantum” field we cannot determine the values of observables. These values can be predicted with probabilities which are determined by the prequantum field, see Chapter ?? for a measurement theory in the PCSFT-framework.

the normalization condition

$$\int_{\mathbf{R}^3} |\phi(x)|^2 = 1, \quad (3)$$

but any vector ϕ in H can be selected as a PCSFT-state. These prequantum waves evolve in accordance with Schrödinger's equation; formally, the only difference is that the initial condition ϕ_0 is not normalized by 1, see Section 2.4, equation (13). Thus, these PCSFT-waves are closely related to Schrödinger's quantum waves. However, opposite to Schrödinger and to the orthodox Copenhagen interpretation, the wave function of the QM-formalism is not a state of a quantum system. In the complete accordance with Einstein's dream of reducibility of quantum randomness, wave function is associated with an ensemble. The ensemble, however, not of quantum systems, but the ensemble of classical fields, or, more precisely, a *classical random field*, random signal. It is appropriate to say that, although our model supports Einstein's views on the origin of quantum randomness, it also matches von Neumann's views [270] on individual quantum randomness. By using ergodicity, see Section ??, we can switch from ensemble description to individual signal description and vice versa. We state again that such a possibility of peaceful combination of Einstein's and von Neumann's views on quantum randomness is a consequence of the rejection of the corpuscular model in the complete accordance with the views of "late Einstein." (It seems that at first he wanted to reduce quantum randomness to randomness of ensembles of particles.)

A random field (at a fixed instant of time) is a function $\phi(x, \omega)$, where ω is a random parameter. Thus for each ω_0 , we obtain the classical field, $x \mapsto \phi(x, \omega_0)$. Another picture of the random field is the H -valued random variable, each fixed ω_0 determines a vector $\phi(\omega_0) \in H$. A random field is given by a probability distribution on H . For simplicity, we can consider a finite-dimensional Hilbert space instead of $L_2(\mathbf{R}^3)$ (as people often do in quantum information theory). In this case, PCSFT considers H -valued random vectors, where $H = \mathbf{C}^n$. (However we strongly emphasize the role of the physical state space $H = L_2(\mathbf{R}^3)$, see also [266], [132].)

This is the ensemble model of the random field. In the rigorous mathematical framework it is based on the *Kolmogorov probability space* [194] $(\Omega, \mathcal{F}, \mathbf{P})$, where Ω is a set and \mathcal{F} is the σ -algebra of its subsets, \mathbf{P} is a probability measure on \mathcal{F} . It is always possible to

choose $\Omega = H$ and \mathcal{F} as the σ -algebra of Borel subsets¹⁹ of H , and probability is a measure on the Hilbert space H . We remark that such measures are used in classical signal theory as probability distributions of random signals.

In the classical signal theory one can move from the ensemble description of randomness to the time series description – under the ergodicity hypothesis, see Section ?? . Random signals are widely used e.g. in radio-physics [239]; these are electromagnetic fields depending of a random parameter; by using the Riemann-Silberstein representation stationary radio-signals can be represented in the complex form: $\phi(x, \omega) = E(x, \omega) + iB(x, \omega)$.

Remark 2.1. Einstein used to make a point that the wave function Ψ is a label for an ensemble of identically prepared quantum systems. However, it was far from clear which statistical characteristics of an ensemble are encoded in Ψ . Obviously, not all of them, since Einstein lamented that QM is not complete. Our model, PCSFT, specifies the statistical characteristics are encrypted in Ψ , these are correlations between components of the field. The correlations are described by the covariance operator of the probability distribution of hidden variables of the field-type. This is an important improvement of the statistical interpretation of QM. Instead of the Einstein’s vague statement (see also Margenau [215] and Ballentine [25]–[28]) about statistical characteristics of an ensemble, we discovered the classical statistical variable, the covariance operator, which was formally used in the QM-formalism under the name “wave function”. Finally, we remark that people using the operational interpretation of QM (e.g., Ludwig, Davis, D’Ariano, Holevo, Busch, Grabowski, Lahti, Ozawa [207], [43], [102], [103], [?], [51], [60] [49], [229], [230]) typically proceed with the ensemble interpretation, too. In contrast to Einstein, Margenau, and Ballentine, they are sure that Ψ encodes all possible statistical characteristics of an ensemble, because they believe in completeness of QM. At the first sight, PCSFT presents a strong argument against such a viewpoint (introducing a new statistical characteristic): the covariance operator does not determine a probability distribution

¹⁹This is the minimal system of subsets H containing all balls in H and closed with respect to countable unions, intersections and complements of sets. In particular, it contains all open and closed sets. However, the reader with the background in physics can relax: in this book we shall never use measure-theoretic constructions at the mathematical level. It is enough to know (without mathematical details) about such notions as measure and integral.

uniquely. Therefore a random field contains essentially more information than given by the covariance operator. However, if a prequantum field is Gaussian, it is completely determined by its covariance operator. (We shall consider only random fields with zero average.) Thus, for Gaussian prequantum fields, views of the adherents to the “orthodox ensemble interpretation” can be easily combined with views of the adherents to the operational approach to QM. (As we see, surprisingly many contradictions between different interpretations of QM can be resolved by PCSFT.)

2.2 Covariance operator interpretation of wave function

Main message: *The wave function is not a field of probabilities or a physical field. It encodes correlations between degrees of freedom of a prequantum random field.*

For simplicity, in this introductory section we consider the case of a single, i.e., noncomposite, system, e.g., the electron (nonrelativistic, since the present PCSFT is a nonrelativistic theory²⁰) and we neglect for a moment (again for simplicity) fluctuations of vacuum which will play an important role in our further consideration.

In our model the wave function Ψ of the QM-formalism encodes a class of prequantum random fields having the same covariance operator (determined by Ψ and determining a unique Gaussian random field.) We state again that we consider the case of a noncomposite quantum system; for composite systems, e.g., for a pair of photons or electrons, the correspondence between the wave function of QM and the covariance operator of PCSFT is more complicated, see Chapter 2.

In this situation the covariance operator (normalized by dispersion) is given by the orthogonal projector on the vector Ψ :

$$D_\Psi = \Psi \otimes \Psi, \quad (4)$$

i.e., $D_\Psi u = \langle u, \Psi \rangle \Psi$, $u \in H$. Thus,

$$D_\Psi = |\Psi\rangle\langle\Psi|$$

²⁰It seems that there are no problems (neither physical nor mathematical) to develop a relativistic variant of PCSFT. I plan to do this in future.

in Dirac's notation, i.e.,

$$D_\Psi u = \langle u | \Psi \rangle | \Psi \rangle.$$

We also suppose that all *prequantum fields have zero average*

$$E\langle y, \phi \rangle = 0, y \in H, \quad (5)$$

where E denotes the classical *mathematical expectation* (average, mean value). By applying a linear functional y to the random vector ϕ we obtain the scalar random variable. In the L_2 -case we get a family of scalar random variables:

$$\omega \mapsto \xi_y(\omega) \equiv \int_{\mathbf{R}^3} \phi(x, \omega) \overline{y(x)} dx, y \in L_2.$$

We recall that the covariance operator D of a random field (with zero average) $\phi \equiv \phi(x, \omega)$ is defined by its bilinear form

$$\langle Du, v \rangle = E\langle u, \phi \rangle \langle \phi, v \rangle, u, v \in H. \quad (6)$$

Under the additional assumption that the prequantum random fields are *Gaussian*, the covariance operator uniquely determines the field. Although this assumption seems to be quite natural both from the mathematical and physical viewpoints, we should be very careful. In the case of a single system we try to proceed as far as possible without this assumption. However, the PCSFT-description of composite systems is based on Gaussian random fields (Section ??, see also Section ?? for general discussion of a possible physical origin of Gaussian probability distributions on the prequantum level.) Let $H = \mathbf{C}^n$ and $\phi(\omega) = (\phi_1(\omega), \dots, \phi_n(\omega))$, then zero average condition (5) is reduced to

$$E\phi_i \equiv \int_{\Omega} \phi_k(\omega) dP(\omega) = 0, k = 1, \dots, n;$$

the covariance matrix $D = (d_{kl})$, where

$$d_{kl} = E\phi_k \bar{\phi}_l \equiv \int_{\Omega} \phi_k(\omega) \overline{\phi_l(\omega)} dP(\omega).$$

We also recall that the dispersion of the random variable ϕ is given by

$$\sigma_\phi^2 = E\|\phi(\omega) - E\phi(\omega)\|^2 = \sum_{k=1}^n E|\phi_k(\omega) - E\phi_k(\omega)|^2.$$

In the case of zero average we simply have

$$\sigma_\phi^2 = E\|\phi(\omega)\|^2 = \sum_{k=1}^n E|\phi_k(\omega)|^2.$$

Here it is always possible to select Ω (the set of random parameters) as \mathbf{C}^n . Then, the above integrals will be over \mathbf{C}^n . In particular, by selecting Gaussian, complex-valued, random fields we obtain Gaussian integrals over \mathbf{C}^n .

The case $H = L_2$ is more complicated from the measure-theoretic viewpoint, since this space is infinite-dimensional. In the case of non-composite systems (i.e., a single photon or electron) it is also possible to select $\Omega = H$, i.e., to integrate with respect to all fields of the L_2 -class. For composite systems, the situation is more complicated. Here we cannot proceed without taking into account the background field, that is of the white noise type. And the well-known fact is that the probability distribution of white noise cannot be concentrated on L_2 , one has to select Ω as a space of distributions, i.e., to integrate with respect to singular fields.

We also remark that the random field $\phi(x, \omega)$ corresponding to a pure quantum state is not L_2 -normalized. Its L_2 -norm

$$\|\phi\|^2(\omega) \equiv \int_{\mathbf{R}^3} |\phi(x, \omega)|^2 dx \quad (7)$$

fluctuates depending on the random parameter ω . We call the quantity

$$\pi_2(\phi) \equiv \|\phi\|^2$$

the *power of the prequantum field (signal)* ϕ . This quantity will play a crucial role in the measurement theory corresponding to PCSFT, see Chapter ??.

We shall distinguish the power of a prequantum field from its energy. The latter is given by the Hamilton function $\mathcal{H}(\phi)$, (functional (16)) which is also a quadratic functional of the prequantum field. However, in contrast to the “pure field dependence” of $\pi_2(\phi)$, the Hamilton function $\mathcal{H}(\phi)$ depends on some parameters (mass, charge, external potential). The $\|\phi\|^2(\omega)$ is the power of the ω -realization of the random field.

2.3 Quantum observables from quadratic forms of the prequantum field

Main message: *In spite of all no-go theorems (e.g., the Kochen-Specker theorem), a natural functional representation of quantum observables exists.*

In PCSFT quantum observables are represented by corresponding quadratic forms of the prequantum field.²¹ A self-adjoint operator \hat{A} is considered as the symbolic representation of the PCSFT-variable

$$\phi \mapsto f_A(\phi) = \langle \hat{A}\phi, \phi \rangle. \quad (8)$$

This is a map from the L_2 -space of classical prequantum fields into real numbers, a quadratic form.

We remark that f_A can be considered as a function on the phase space of classical fields: $f_A \equiv f_A(q, p)$, where $\phi(x) = q(x) + ip(x)$, i.e., it is possible to move from the complex representation to the phase space representation and vice versa, see Chapter 3. A crucial point is that the *prequantum phase space* is infinite-dimensional (and the “post-quantum phase space”, i.e., the phase space of ordinary classical mechanics is finite-dimensional).

The average of this quadratic form with respect to the random field determined by the wave function Ψ coincides with the corresponding quantum average:

$$\langle f_A \rangle = \langle \hat{A}\Psi, \Psi \rangle \quad (9)$$

or

$$\langle f_A \rangle = \langle \Psi | \hat{A} | \Psi \rangle$$

in Dirac’s notation. Here

$$\langle f_A \rangle = Ef_A(\phi) = \int_H f_A(\phi) d\mu_\Psi(\phi)$$

is the classical average and μ_Ψ is the probability distribution of the prequantum random field $\phi \equiv \phi_\Psi$ determined by the pure quantum state Ψ . In the real physical case H is infinite-dimensional; the classical average is given by the integral over all possible classical fields; probabilistic weights of the fields are determined, in general, non-uniquely,

²¹This is true for a part of PCSFT reproducing QM, cf. Chapter 2, Section ?? and Chapter ?? for the PCSFT “beyond quantum model”.

by the Ψ . Thus, the quantum formula for the average of an observable was demystified:

$$\langle \hat{A} \rangle_{\Psi} \equiv \langle \hat{A} \Psi, \Psi \rangle = \int_H f_A(\phi) d\mu_{\Psi}(\phi) \quad (10)$$

It can be obtained via the classical average procedure.

2.4 Quantum and prequantum interpretations of Schrödinger's equation

Main message: *Schrödinger's equation with random initial conditions describes dynamics of the physical random field.*²²

Before going to the PCSFT-dynamics, we consider the Schrödinger equation in the standard QM-formalism:

$$i\hbar \frac{\partial \Psi}{\partial t}(t, x) = \hat{\mathcal{H}} \Psi(t, x), \quad (11)$$

$$\Psi(t_0, x) = \Psi_0(x), \quad (12)$$

where $\hat{\mathcal{H}}$ is Hamiltonian, the energy observable. We recall that Schrödinger tried to interpret $\Psi(t, x)$ as a classical field (e.g., the electron field; the distribution of electron charge in space). However, he gave up and, finally, accepted the conventional interpretation, the probabilistic one, due to Max Born.

We recall that a time dependent random field $\phi(t, x, \omega)$ is called a *stochastic process* (with the state space $H = L_2$). Dynamics of the prequantum random field is described by the simplest stochastic process which is given by *deterministic dynamics with random initial conditions*.

In PCSFT the Schrödinger equation, but with the random initial condition, describes dynamics of the prequantum random field, i.e., the prequantum stochastic process can be obtained from the mathematical equation which is used in QM for dynamics of the wave function:

$$i\hbar \frac{\partial \phi}{\partial t}(t, x, \omega) = \hat{\mathcal{H}} \phi(t, x, \omega), \quad (13)$$

$$\phi(t_0, x, \omega) = \phi_0(x, \omega), \quad (14)$$

²²In the biparticle case Schrödinger's equation describes dynamics of the two-points correlation function for field components, see Section ??.

where the initial random field $\phi_0(x, \omega)$ is determined by the quantum pure state Ψ_0 . Standard QM gives the covariance operator of this random field.

Roughly speaking, we combined Schrödinger's and Born's interpretations: the Ψ -function of QM is not a physical field, but for each t it determines a random physical field, i.e., the H -valued stochastic process $\phi(t, x, \omega)$.

PCSFT dynamics matches standard QM-dynamics by taking into account the PCSFT-interpretation of the wave-function, see (4). Denote by $\rho(t)$ the covariance operator of the random field $\phi(t) \equiv \phi(t, x, \omega)$, the solution of (13), (14). Then

$$\rho(t) \equiv \rho_{\Psi(t)} = \Psi(t) \otimes \Psi(t),$$

where $\Psi(t)$ is a solution of (11), (12).

Such simple description can be used only for a single system and in the absence of fluctuations of vacuum. In the general case of a composite system, e.g., a biphoton system, in the presence of vacuum fluctuations Schrödinger dynamics of the Ψ -function encodes only dynamics of the covariance operator of the prequantum stochastic process, see Chapter 2, Section ???. The situation is essentially more complicated than in the case of a single system. We found that it is possible to construct a few different prequantum dynamics which match (on the level of correlations) QM-dynamics, see Section ???.

2.5 Towards prequantum determinism?

Main message: *The background field is everywhere.*

From the PCSFT-viewpoint, the source of quantum randomness is the randomness of initial conditions (if one neglects vacuum fluctuations), i.e., impossibility to prepare a non-random initial prequantum field $\phi_0(x)$.

We expect that in future very stable and precise preparation procedures will be created. The output of such a procedure will be a deterministic field $\phi(x)$, i.e., random fluctuations will be eliminated.

However, this dream of creating supersensitive “subquantum” technologies which would recover determinism on the microlevel may never come true. In such a case PCSFT will play the role of classical statistical mechanics of prequantum fields²³. Unfortunately, there are a few

²³In ordinary classical statistical mechanics the existence of Hamiltonian dynamics has

signs that it really might happen. First of all, it might be that the scale of prequantum fluctuations is very fine, e.g., the *Planck scale*. In this case it would be really impossible to prepare a deterministic prequantum field. And there is another reason. The PCSFT-model presented up to now has been elaborated for noncomposite quantum systems, e.g., a single electron. The extension of PCSFT to composite systems, e.g., a pair of entangled photons or electrons, is based on a more complicated model of prequantum randomness, see Chapter 2, Section ??.

We should complete the present model by considering fluctuations of the background field (zero point field, vacuum fluctuations), in the same way as in SED. In reality, these are always present. Therefore Einstein's dream of determinism cannot be peacefully combined with the presence of the background field. If this field is irreducible (as a fundamental feature of space), then deterministic prequantum fields will never be created.²⁴ However, if this background field is simply noise²⁵ which can be eliminated, then we can dream of the creation of deterministic prequantum fields. However, a possibility to prepare such fields does not imply deterministic reduction of QM. As was already pointed out, the inter-relation between prequantum fields and quantum observables given by TSD (measurement theory of classical waves with threshold detectors) is really tricky, see Section ??.

2.6 Random fields corresponding to mixed states

Main message: *A density matrix is the normalized covariance operator of a prequantum random field.*

We now consider the general quantum state given by a density operator ρ . (We still work with noncomposite quantum systems.) According to PCSFT, ρ determines the covariance operator of the corresponding prequantum field (under normalization by its dispersion)

$$D_\rho = \rho. \quad (15)$$

merely a theoretical value. In real applications we operate with probability distributions on the phase space. Corresponding dynamics is described by Liouville equation.

²⁴As I understood from conversations with Gerard 't Hooft, in his model [260]–[262] a background random field which is considered as fluctuations of space-time by itself also plays an important role. Nevertheless, he claims that at the subquantum level determinism can be completely restored. How?

²⁵Hence the completely empty physical space can be really prepared, “distilled from noise”.

Dynamics of the corresponding prequantum field $\phi(t, x, \omega)$ is also described by the Schrödinger equation, see (13), (14), with the random initial condition $\phi_0(x, \omega)$. The initial random field has the probability distribution μ_{ρ_0} having zero mean value and the covariance operator

$$D(t_0) = \rho_0.$$

Under the assumption that all prequantum random fields are Gaussian, the initial probability distribution is determined in the unique way. In the general (non-Gaussian) case we lose the solid ground. The $\phi_0(x, \omega)$ can be selected in various ways, i.e., it can be any distribution having the covariance $D(t_0)$. We could not exclude such a possibility. It would simply mean that macroscopic preparation procedures are not able to control even the probability distribution (only its covariance operator).

Denote by $\rho(t)$ the covariance operator of the random field $\phi(t) \equiv \phi(t, x, \omega)$ given by (13), (14) with ϕ_0 having the covariance operator $\rho(t_0) = \rho_0$. Then $\rho(t)$ satisfies the von Neumann equation. However, $\rho(t)$ has the classical probability interpretation as the covariance operator $D(t)$. In the Gaussian case $D(t)$ determines completely the prequantum probability distribution.

2.7 Background field

Main message: *QM is a formalism of measurement with calibrated detectors (filtering vacuum fluctuations).*

In the general PCSFT-framework the randomness of the initial conditions has to be completed by taking into account fluctuations of vacuum (to obtain a consistent PCSFT which works both for one particle system and biparticle system). In our model the background field (vacuum fluctuations) is of the white noise type. It is a Gaussian random field with zero average and the covariance operator

$$D_{\text{background}} = \varepsilon I, \quad \varepsilon > 0.$$

It is a stationary field, so its distribution does not change with time.

Consider (by using the QM-language) a quantum system in the mixed state ρ_0 . It determines the prequantum random field $\phi_0 \equiv \phi_0(x, \omega)$ with the covariance operator

$$\tilde{D}(t_0) = \rho_0 + \varepsilon I.$$

The value of $\varepsilon > 0$ is not determined by PCSFT, but it could not be too small for a purely mathematical reason, see Chapter ??, Section ??. (Hence QM is a theory of filtration of strong noise.) Now consider the solution $\phi(t)$ of the Schrödinger equation (13), (14) with the initial condition ϕ_0 . Its covariance operator can be easily found:

$$\tilde{D}(t) = D(t) + \varepsilon I,$$

where $D(t)$ is the covariance operator of the process in the absence of the background field, $D(t) = \rho(t)$. Here $\rho(t)$ satisfies the QM-equation for evolution of the density operator, i.e, the von Neumann equation. Thus on the level of dynamics of the covariance operator the contribution of the background field is very simple: an additive shift. However, on the level of the field dynamics the presence of vacuum fluctuations changes the field behavior crucially.

Consider the prequantum random field $\phi_0(x, \omega)$ corresponding to a pure quantum state Ψ_0 . Now (in the presence of the background field) the prequantum random field $\phi_0(x, \omega)$ is not concentrated on a one-dimensional subspace²⁶

$$H_{\Psi_0} = \{\phi = c\Psi_0 : c \in \mathbf{C}\};$$

the vacuum fluctuations smash it over H .

In the canonical QM the background field of the white noise type is neglected; in fact, it is eliminated “by hand” in the process of detector calibration. And it is the right strategy for a formalism describing measurements on the random background. However, in an ontic model, i.e., a model of reality as it is, this background field should be taken into account. Neglecting it induces a rather mystical picture of quantum randomness.

We shall see that in the PCSFT-formalism the background field plays the fundamental role in the derivation of Heisenberg’s uncertainty relation, see Section ??. Roughly speaking, Heisenberg’s uncertainty is a consequence of vacuum fluctuations.²⁷

²⁶In the absence of vacuum fluctuations the covariance operator of the random field $\phi_\Psi(x, \omega)$ corresponding to a pure state Ψ is given by the orthogonal projector on Ψ , see (4); the corresponding Gaussian measure is concentrated on a one-dimensional subspace generated by Ψ . Of course, the latter is valid only for Gaussian prequantum fields.

²⁷A similar viewpoint on Heisenberg’s uncertainty relation can be found in Hofmann’s PhD thesis [101] (1999).

2.8 Coupling between Schrödinger and Hamilton equations

Main message: *The Schrödinger equation is a complex form of the Hamilton equation for a special class of quadratic Hamilton functions on an infinite-dimensional phase space.*

We remark that the Schrödinger equation can be written as the system of Hamilton equations on the (infinite-dimensional) phase space $Q \times P$, where $Q = P$ is the real Hilbert space and $H = Q \oplus iP$ is the corresponding complex Hilbert space. The prequantum field $\phi(x) = q(x) + ip(x)$, where $q(x)$ and $p(x)$ are real-valued fields (or more generally, they take values in \mathbf{R}^m). Consider the Hamilton function

$$\mathcal{H}(q, p) = \frac{1}{2} \langle \widehat{\mathcal{H}} \phi, \phi \rangle, \quad (16)$$

or, in Dirac's notation,

$$\mathcal{H}(\phi) = \frac{1}{2} \langle \phi | \widehat{H} | \phi \rangle.$$

see Chapter 3 for details; in PCSFT $\mathcal{H}(q, p)$ is the energy of the prequantum field $\phi(x) = q(x) + ip(x)$. The Schrödinger equation (13) can be written as the system

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}, \quad \dot{p} = -\frac{\partial \mathcal{H}}{\partial q}, \quad (17)$$

see Strochi [258]. From the PCSFT viewpoint, there is no reason (at least mathematical) to use only quadratic Hamiltonian functions. By considering non-quadratic Hamilton functions we obtain Hamilton systems connected with the nonlinear Schrödinger equation, cf. [31], [273], [82], [83], [64], [65]. PCSFT naturally induces a nonlinear extension of QM, see Sections ?? and ??.

2.9 Nonquadratic functionals of the prequantum field and violation of Born's rule

Main message: *Nonlinear, of order higher than two, contribution of the prequantum field induces violation of Born's rule.*

In principle, there is no reason to restrict PCSFT-variables to quadratic functionals of the prequantum fields, see (9). Let us consider an arbitrary smooth functional $f(\phi), \phi \in H$, which maps the

field $\phi \equiv 0$ into zero, $f(0) = 0$. Let us also consider a random field $\phi = \phi(x, \omega)$ corresponding to a quantum density operator ρ . We can find the classical average

$$\langle f \rangle_\mu = \int_H f(\phi) d\mu(\phi), \quad (18)$$

where μ is the probability distribution of the random field. We shall show, see Chapter ??, that this classical average can be approximated by the quantum average

$$\langle \hat{A} \rangle_\rho = \text{Tr} \rho \hat{A}, \quad (19)$$

where

$$\hat{A} = f''(0)/2 \quad (20)$$

is the second derivative of the field functional $f(\phi)$ at the point $\phi = 0$ (divided by the factor 2 which arises from the Taylor expansion). If a Hilbert state space is finite-dimensional, then this is the usual second derivative. Its matrix (*Hessian*) is symmetric. If a Hilbert space is infinite-dimensional (of the L_2 -type), then the derivatives are so-called variations. In the rigorous mathematical framework they are Frechet derivatives, that are used, e.g., in optimization theory. In the latter case the second (variation) derivative is given by a self-adjoint operator. This is the PCSFT-origin of the representation of quantum observables by self-adjoint operators.

Quantum observables are represented by self-adjoint operators, since they correspond to Hessians of smooth functionals of the prequantum field.

Thus the QM-formalism gives approximations of classical averages with respect to the prequantum random fields by approximating field-functionals $f(\phi)$ by the quadratic terms of their Taylor expansions.

If the functional $f(\phi)$ is linear, $f(\phi) = \langle \phi, y \rangle, y \in H$, then its QM-image, the second derivative, is equal to zero. Linear field effects are too weak and they are completely ignored by QM. However, such functionals and their correlations are well described by PCSFT. Observation of such effects can be the first step beyond QM, see Section ??.

2.10 Wave comeback – a solution too cheap?

Main message: *Physical space exists!*²⁸ Hence waves propagating in this space are basic entities of nature.

It is well known that Einstein was not happy with Bohmian mechanics. He considered this solution of the problem of completion of QM as cheap. Recently Anton Zeilinger mentioned (in his lecture at the Växjö conference-2010, “Advances in Quantum theory”) that QM may be not the last theory of micro processes and in future a new fundamental theory may be elaborated. And looking me in the face, he added that those who nowadays criticize QM and dream of a prequantum theory will be terrified by this coming new theory, by its complexity and extraordinariness. They will recall the old QM-formalism, i.e., the present one, with great pleasure, since it was so close to classical mechanics. A similar viewpoint on a coming prequantum theory was presented by Claudio Garola during our dialog on possible ways to proceed beyond QM [79].

PCSFT is a comeback to classical field theory; roughly speaking, in the spirit of early Schrödinger and late Einstein: the Maxwell classical field theory is extended to “matter waves”. Of course, this comeback is not the dream of the majority of those who nowadays are not afraid to speculate on prequantum models and criticize the Copenhagen QM. Nevertheless, I do not think that PCSFT is a cheap completion of the standard QM. I hope that, in contrast to Bohmian mechanics, Einstein might accept PCSFT as one of the possible ways beyond QM. In any event the Laplacian mechanistic determinism was totally excluded from PCSFT; reality became blurred in the sense of Schrödinger [247], [248]. This is reality of fields and not particles, but still reality.

3 Where is discreteness? Devil in detectors?

Prequantum variables $f_A(\phi) = \langle \hat{A}\phi, \phi \rangle$ have continuous ranges of values. On the other hand, in QM some observables have discrete spectra. Thus, although PCSFT matches precisely probabilistic predictions of

²⁸By this statement Igor Volovich has started his talks at Växjö conferences for ten years criticizing quantum information theory which practically ignores this fact.

QM²⁹, it violates the *spectral postulate* of QM. How can one obtain discrete spectra?

The continuous field model supports the viewpoint that “ontic reality”, i.e., reality as it is, is continuous.³⁰ *Discreteness of some observable data is created by our macroscopic devices* which split a prequantum signal in a number of discrete channels. Take a polarization beam splitter (PBS). Consider first a classical signal. Suppose that PBS is oriented at an angle θ . Then the classical signal is split into two channels. We can label these channels as “polarization up”, $S_\theta = +1$, and “polarization down”, $S_\theta = -1$, (for θ -direction). The only problem is that the classical signal is present in both channels. Thus we cannot assign to a classical signal (even to a short pulse) a concrete value of S_θ . On the contrary, for a “quantum signal” (photon), detectors never click in both channels; we get either $S_\theta = +1$ or $S_\theta = -1$. This is a standard example of quantum discreteness.

The first comment of this common description is that the situation “no double clicks” is never occurred in real experiments, see e.g., [22], [84], [85], [11], [9]. There are always *double clicks!* And they are many! They are partially discarded by using the *time window*. However, this is just a remark on the standard measurement procedure. The main point is that it is possible to produce discrete clicks even from a classical continuous signal by using threshold-type detectors. My PhD-student Guillaume Adenier performed numerical simulation for the threshold detection model of classical signals. He reproduced quantum probabilities of detection and even in a more complicated framework of classical bi-signals interacting with two PBSs oriented at angles θ_1 and θ_2 the EPR-Bohm correlations; Bell’s inequality was violated, see [13].

²⁹In fact, the situation is more complicated. By considering quadratic prequantum variables $f_A(\phi)$ we obtain the coincidence of prequantum classical and quantum averages, see Chapters 2,3. However, by considering nonquadratic functionals of prequantum fields we find that the quantum probability given by Born’s rule is just the main contribution to the prequantum (classical) average.

³⁰In fact, my viewpoint on a proper mathematical model of reality is more complicated. Of course, the usage of continuous space-time based on real numbers is just a way to unify a huge hierarchy of scales of space and time. In this book we do not criticize this model, cf. with, e.g., p -adic models, Vladimirov, Volovich, Witten, Freund, Dragovich, Aref’eva, Frampton, Parisi, Khrennikov, Zelenov, Kozyrev, see, e.g., [265], [118], [119]. At the moment we “just” criticize Bohr’s postulate, the existence of the fundamental quant of action given by the Planck constant. We predict splitting of values of quantum observables at finer (“prequantum”) time scales.

In particular, according to our model, electromagnetic field is quantized only in the process of interaction with matter. This viewpoint matches well views of Lamb [198], Lande [199], [200], Kracklauer [195], [196], Roychoudhuri [242], [243], Adenier [13], people working in SED, e.g. Marshall and Brafford, Boyer, de la Pena, Ceto, Coli, ... [41], [53]–[57], [46]. However, PCSFT differs essentially from a rather popular idea that the electromagnetic field is continuous, but matter is quantized. This viewpoint was stressed in the books of Lande [199], [200]). PCSFT does not quantize even the matter, the latter also consists of continuous fields fluctuating on very fine space-time scales. These scales are not yet approachable. In future we expect to get a possibility to monitor these fields and not only their averaged images given by quantum particles. SED-like people do not expect this. It seems that only Albert Einstein might be happy with PCSFT.

4 On experiments to tests the Euclidean model

One of the most famous stories about Gauss depicts him measuring the angles of the great triangle formed by the mountain peaks of Hohenhagen, Inselberg, and Brocken for evidence that the geometry of space is non-Euclidean. He tested the inequality:

$$\alpha_{12} + \alpha_{23} + \alpha_{13} < 2\pi, \quad (21)$$

where α_{ij} is the angle between the corresponding sides of the triangle. Gauss understood how the intrinsic curvature of the Earth's surface would theoretically result in slight discrepancies when fitting the smaller triangles inside the larger triangles, although in practice this effect is negligible, because the Earth's curvature is so slight relative to even the largest triangles that can be visually measured on the surface. Still, Gauss computed the magnitude of this effect for the large test triangles because, as he wrote, “the honor of science demands that one understand the nature of this inequality clearly”.

On the other hand, if the curvature of space was actually great enough to be observed in optical triangles of this size, then presumably Gauss would have noticed it, so we may still credit him with having performed an empirical observation of geometry, but in this sense every person who ever lived has made such observations. The first person to publicly propose an actual test of the geometry of space

was apparently Lobachevsky, who suggested that one might investigate a stellar triangle for an experimental resolution of the question. The stellar triangle he proposed was the star Sirius and two different positions of the Earth at 6-month intervals. This was used by Lobachevsky as an example to show how we could place limits on the deviation from the flatness of actual space based on the fact that, in a hyperbolic space of constant curvature, there is a limit to how small a star's parallax can be, even for the most distant star. The first definite measurement of the parallax for a fixed star was performed by Friedrich Bessel (a close friend of Gauss') in 1838, on the star 61 Cygni. Shortly thereafter he measured Sirius (and discovered its binary nature). Lobachevsky's first paper on the new geometry was presented as a lecture in Kasan in 1826 followed by publications in 1829, 1835, 1840, and 1855 (a year before his death). He presented his lower bound for the characteristic length of a hyperbolic space in the later editions based on the still fairly recent experimental results of stellar parallax measurements.

The inequality

$$\alpha_{12} + \alpha_{23} + \alpha_{13} = 2\pi \quad (22)$$

is a geometric analog of Bell's inequality. Violation of (22), e.g., in the form of (21) implies impossibility to use the Euclidean model. In the same way violation of Bell's inequality implies impossibility to use the Kolmogorov model.

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